tion of the curve of the isotherm path and the x axis; K, temperature coefficient; a, diffusivity; T_w, T^*, temperature of surface of material and isotherm under consideration; T_0, temperature of unheated material; V_∞, quasisteady value of surface velocity; KT_p, material disintegration constant; $S(\tau)$, linear ablation; q_c, calorimetric heat flux; I_e, stagnation enthalpy; q'_{mn}, mean integral heat flux during the period from 0 to τ_T; λ, thermal conductivity; ρ, density; c, heat capacity; m, parameter of thermal efficiency of the material; δ_T, depth of heating.

LITERATURE CITED

1. Yu. V. Polezhaev, V. P. Romanchenkov, I. V. Chirkov, and V. N. Shebeko, Inzh.-Fiz. Zh., 37, No. 3, 395-404 (1979).
2. Yu. V. Polezhaev, Inzh.-Fiz. Zh., 37, No. 3, 390-394 (1979).
3. Yu. A. Tadol'der, Tr. Tallin. Politekh. Inst. Ser. A, No. 223 (1966), pp. 3-13.
4. G. A. Frolov, V. V. Pasichnyi, Yu. V. Polezhaev, and A. V. Choba, Inzh.-Fiz. Zh., 52, No. 1, 33-37 (1987).
5. G. A. Frolov, A. A. Korol', V. V. Pasichnyi, et al., Inzh.-Fiz. Zh., 51, No. 6, 932-940 (1986).
6. Yu. V. Polezhaev and F. B. Yurevich, Thermal Protection [in Russian], Moscow (1976).
7. G. A. Frolov, Inzh.-Fiz. Zh., 50, No. 4, 629-635 (1986).
8. O. A. Sergeev, A. G. Shashkov, and A. S. Umanskii, Inzh.-Fiz. Zh., 43, No. 6, 960-970 (1982).

EXPERIMENTAL VERIFICATION OF MODELS OF TRIPLE MIXED CORRELATION BETWEEN VELOCITY AND TEMPERATURE AS APPLIED TO THE CALCULATION OF JET FLOWS

Yu. M. Dmitrenko, V. L. Zhdanov, N. N. Luchko, and P. Ya. Cherepanov

UDC 532.517.4

The optimum values of the constants in the well-known approximation of the moment $\overline{u_2^2\theta}$ are determined from the results of experimental investigations of nonisothermal wakes.

Multiparameter differential $\overline{u_iu_j}-\varepsilon_u-\overline{u_i\theta}-\overline{\theta^2}-\varepsilon_\theta$ models of turbulence are now being used ever more extensively in practical engineering calculations. They are suitable for calculating a broad class of complicated flows and enable one to obtain information about the pulsation characteristics, which is important in a number of technical applications (e.g., in problems of the propagation of radiation in a turbulized medium, in the construction of mixing devices, the calculation of thermal stresses in heat-releasing elements arising from the presence of temperature pulsations in the oncoming stream, etc.).

It is obvious that the reliability of the calculation depends on the soundness of the closing hypotheses for the unknown moments, each of which requires careful verification. In fact, a situation is possible when the individual terms of the equations of turbulent transport are roughly modeled but the end result of the calculation is still found to be close to the experimental data. But if there is no agreement, it is unclear just which hypotheses laid at the foundation of the model do not accord with reality.

The results of an experimental test of various approximations used in models of momentum transfer are given, in particular, in [1-3]. The dynamics of the turbulent field of a passive admixture (temperature) has been studied in less detail, however, both theoretically and experimentally. The development of experimental technique and the surmounting of a number of methodological difficulties now make it possible to measure different mixed moments of velocity and temperature pulsations and thereby estimate the reliability of approximations

A. V. Lykov Institute of Heat and Mass Exchange, Academy of Sciences of the Belorussian SSR, Minsk.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 3, pp. 362-369, March, 1987. Original article submitted February 11, 1986.

0022-0841/87/5203-0259$12.50

of certain terms in the model equations of turbulent heat transfer. The results of testing the well-known relations for the triple moments $\overline{u_i u_j \theta}$ and $\overline{u_i \theta^2}$ are contained in [4, 5]. In both cases, however, the results of an experimental investigation of only one type of flow were used: a plane, asymmetric heated jet [4] and an axisymmetric thermal wake with a zero excess impulse [5], and only the approximation for $\overline{u_2 \theta^2}$ was tested in the latter paper. Although a comparative analysis of the known approximations is given in these papers and the values of the respective numerical coefficients are determined, it remains unclear whether the advantages revealed for one model or another are retained in the case of other wake flows and whether the empirical constants are universal.

Approximations for the moment $\overline{u_i \theta^2}$, which describe turbulent diffusion in the equation for the intensity of temperature pulsations $\overline{\theta^2}$, were analyzed in [6] from the results of several experiments. In a number of cases, such as in flow with a symmetric velocity field and an asymmetric temperature field, the turbulent heat flux $\overline{u_i \theta}$ depends not only on the local parameters at the given point but also on the entire past history of the flow. The calculation of such nonequilibrium streams requires that the model system for $\overline{T} - \overline{\theta^2} - \varepsilon_\theta$ be supplemented by the transfer equation for $\overline{u_i \theta}$, which contains the moments $\overline{u_i u_j \theta}$.

In the present paper we test the well-known model relations for $\overline{u_i u_j \theta}$ against the results of experimental investigations of three nonisothermal wake flows. In the first two cases the turbulence characteristics were measured in the wakes behind an ellipsoid of revolution with an elongation of 6:1 for different values of the excess impulse J,

$$J = 2\pi\rho \int_0^\infty \overline{U}_1 (\overline{U}_1 - U_\infty)\, x_2 dx_2. \tag{1}$$

A jet of heated air escaped from an opening in the stern region of the ellipsoid. For a low jet velocity a wake regime of flow ($J < 0$) occurred, the characteristics of which are described in [7]. In the second case the jet velocity was chosen from the condition $J = 0$. It is well known that flow with a zero excess impulse develops according to laws differing considerably from the case of $J \neq 0$ [8]. The third experiment differed from the first two not only in the flow geometry but also in the means of creating the temperature inhomogeneity. The wake behind an elongated, plane, symmetric body, propagating in a medium with a linear vertical temperature distribution, was investigated in it. The required temperature profile in the oncoming stream was produced by an electrically heated grid of round rods mounted in the forechamber of a wind tunnel with an enclosed working section [9]. The characteristics of the velocity field in the wake are symmetric and were described in [10]. At the same time, it is well known [11] that turbulent mixing in the wake region introduces antisymmetric disturbances into the temperature profile. In this case, in contrast to shear flows with symmetric temperature distributions, the temperature defect in the plane of symmetry of the wake ($x_2 = 0$) vanishes, while the pulsation heat flux $\overline{u_2 \theta}$ does not vanish. In all the experiments the initial temperature differences were chosen such as to eliminate the influence of buoyant forces on the velocity field, which allowed us to treat heat as a passive admixture. The turbulence characteristics were determined using a 55D thermoanemometric instrument of the DISA ELECTRONIK Company. The mixed moments of velocity and temperature pulsations were measured with a three-filament probe using digital methods of computer analysis of the signals of the primary transducers [12]. The model relations were tested against the results of measurements in cross sections located at sufficiently large distances from the bodies over which the flow occurs ($20 \leq x_1/d \leq 50$), where the errors due to the finite measurement volume of the three-filament probe were relatively small. The dependences of certain characteristics used in the model relations being tested on the transverse dimensionless coordinate $\eta = x_2/\delta_\theta$ are given in Fig. 1. The half-width of the wake was determined from the profile of the temperature pulsations from the condition $\overline{\theta^2}(\delta_\theta) = 0.25\overline{\theta_0^2}$. For convenience in combining different characteristics on one graph, each of them was normalized to its maximum value in the cross section under consideration. These values are given in Table 1.

The above-noted fundamental differences between the characteristics of the velocity and temperature fields in the flows under consideration serve as a good basis for a comprehensive test of the existing approximations for the unknown terms of the heat-transfer equations.

The well-known algebraic expressions used to model the moment $\overline{u_i u_j \theta}$ are given in [4]. We write them in order of increasing complexity:

$$\overline{u_i u_k \theta} = \alpha_u \frac{q^2}{\varepsilon_u} \overline{u_k u_e} \frac{\partial \overline{u_i \theta}}{\partial x_e}, \tag{2}$$

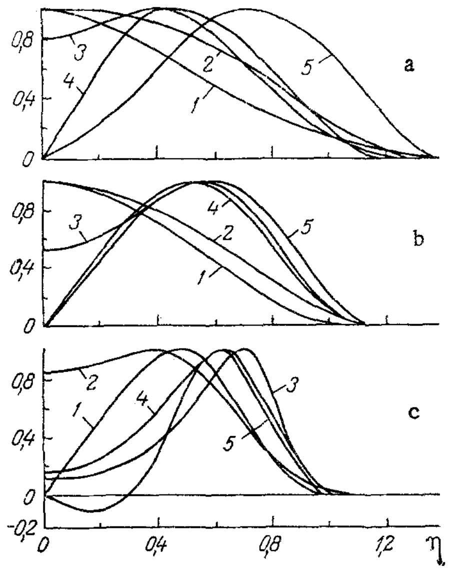

Fig. 1. Distributions of turbulence characteristics across a wake: a) axisymmetric wake ($J < 0$, $x_1/d = 50$); b) axisymmetric wake ($J = 0$, $x_1/d = 20$); c) plane wake ($x_1/d = 41$); 1) $\Delta T/\Delta T_m$; 2) $\overline{u_2^2}/\overline{u_{2m}^2}$; 3) $\overline{\theta^2}/\overline{\theta^2}_m$; 4) $\overline{u_2\theta}/(\overline{u_2\theta})_m$; 5) $\overline{u_2^3}/\overline{u_{2m}^3}$.

$$\overline{u_iu_k\theta} = \alpha_{1u}\frac{q^2}{\varepsilon_u}\left(\overline{u_ku_e}\frac{\partial\overline{u_i\theta}}{\partial x_e} + \overline{u_iu_e}\frac{\partial\overline{u_k\theta}}{\partial x_e}\right), \tag{3}$$

$$\overline{u_iu_k\theta} = \alpha_{2u}\frac{q^2}{\varepsilon_u}\left(\overline{u_ku_e}\frac{\partial\overline{u_i\theta}}{\partial x_e} + \overline{u_iu_e}\frac{\partial\overline{u_k\theta}}{\partial x_e} + \overline{u_e\theta}\frac{\partial\overline{u_iu_k}}{\partial x_e}\right), \tag{4}$$

$$\overline{u_iu_k\theta} = \alpha_{3u}\frac{q^2}{\varepsilon_u}\left(\overline{u_ku_e}\frac{\partial\overline{u_i\theta}}{\partial x_e} + \overline{u_iu_e}\frac{\partial\overline{u_k\theta}}{\partial x_e} + \overline{u_e\theta}\frac{\partial\overline{u_iu_k}}{\partial x_e} + \overline{u_iu_ku_e}\frac{\partial\overline{T}}{\partial x_e}\right) \tag{5}$$

From a comparison of Eqs. (2) and (3) it follows that they coincide for $i = k$ and $\alpha_u = 2\alpha_{1u}$. At the same time, the right side of Eq. (2) does not satisfy the condition of invariance relative to permutation of the indices, so that we shall not consider Eq. (2) further. The factor q^2/ε_u, representing the time scale of the velocity field, appears in each of the approximations. The use of the analogous scale $\overline{\theta^2}/\varepsilon_\theta$ of the temperature field is equally justified. Therefore, along with Eqs. (3)-(5) we shall consider approximations obtained by replacing q^2/ε_u by $\overline{\theta^2}/\varepsilon_\theta$.

A detailed test of each of the approximations (with a different combination of indices) would require making rather laborious and complicated measurements of the second and third moments. In those cases when the boundary-layer approximation is valid, the determining role is played by the transverse pulsation heat flux $\overline{u_2\theta}$ and hence by the mixed moment $\overline{u_2^2\theta}$. Starting from the general expressions (3)-(5) and using the dynamic or the thermal time scale in them, we write the model relations for $\overline{u_2^2\theta}$:

$$\overline{u_2^2\theta} = -2\alpha_{1u}\frac{q^2}{\varepsilon_u}\overline{u_2u_2}\frac{\partial\overline{u_2\theta}}{\partial x_2}, \tag{6}$$

$$\overline{u_2^2\theta} = -2\alpha_{1\theta}\frac{\overline{\theta^2}}{\varepsilon_\theta}\overline{u_2u_2}\frac{\partial\overline{u_2\theta}}{\partial x_2}, \tag{7}$$

$$\overline{u_2^2\theta} = -\alpha_{2u}\frac{q^2}{\varepsilon_u}\left(2\overline{u_2u_2}\frac{\partial\overline{u_2\theta}}{\partial x^2} + \overline{u_2\theta}\frac{\partial\overline{u_2u_2}}{\partial x_2}\right), \tag{8}$$

$$\overline{u_2^2\theta} = -\alpha_{2\theta}\frac{\overline{\theta^2}}{\varepsilon_\theta}\left(2\overline{u_2u_2}\frac{\partial\overline{u_2\theta}}{\partial x_2} + \overline{u_2\theta}\frac{\partial\overline{u_2u_2}}{\partial x_2}\right), \tag{9}$$

$$\overline{u_2^2\theta} = -\alpha_{3u}\frac{q^2}{\varepsilon_u}\left(2\overline{u_2u_2}\frac{\partial\overline{u_2\theta}}{\partial x_2} + \overline{u_2\theta}\frac{\partial\overline{u_2u_2}}{\partial x_2} + \overline{u_2^3}\frac{\partial\overline{T}}{\partial x_2}\right), \tag{10}$$

$$\overline{u_2^2\theta} = -\alpha_{3\theta}\frac{\overline{\theta^2}}{\varepsilon_\theta}\left(2\overline{u_2u_2}\frac{\partial\overline{u_2\theta}}{\partial x_2} + \overline{u_2\theta}\frac{\partial\overline{u_2u_2}}{\partial x_2} + \overline{u_2^3}\frac{\partial\overline{T}}{\partial x_2}\right). \tag{11}$$

TABLE 1. Characteristic Parameters of the Investigated Flows

Type of wake flow	x_1/d	ΔT_m, °C	u'_{2m}, m/sec	θ'_m, °C	$\overline{u_2\theta_m}\cdot 10^3$, m·°C/sec	$\overline{u^3_{2m}}\cdot 10^3$, m³/sec³	δ_θ, mm
Axisymmetric ($J<0$)	20	0,53	0,135	0,11	5,88	1,05	36
Axisymmetric ($J=0$)	50	1,47	0,236	0,35	35,1	5,13	25,6
Plane ($J<0$)	41	0,46	0,408	0,178	34	30,3	27,4

The conformity of these approximations to test data was tested as follows. Taking the coefficient α equal to unity, the right sides of Eqs. (6)-(11) were calculated from the results of measurements of the turbulence characteristics. The experimental profiles were approximated by analytic curves to find the derivatives. In all cases the mean square deviation of the experimental values from the analytic curves did not exceed 2%, which is far less than the measurement error. From the condition of a minimum of the mean square deviation

$$\sigma^2 = \frac{1}{n}\sum_{k=1}^{n}(\overline{u_2^2\theta}_e - \overline{u_2^2\theta}_c)^2$$

of the experimentally found values of the moment $\overline{u_2^2\theta}$ from the calculated values we determined the optimum values of the numerical coefficients α in each of Eqs. (6)-(11).

A comparison of the calculated values of the dimensionless moment $\overline{u_2^2\theta}/(\overline{u_2^2\theta})_m$ with the directly measured values is illustrated in Fig. 2. For clarity, the curves constructed from the approximations containing the dynamic time scale (q^2/ε_u) are given in Fig. 2, I, while those for the thermal time scale $(\theta^2/\varepsilon_\theta)$ are given in Fig. 2, II. As the approximations become more complicated, the deviation of the profiles calculated using them from the experimental data decreases.

Only the models (10) and (11), containing terms proportional to the average temperature gradient, provide acceptable agreement between the calculated and experimental values. At the same time, with the proper choice of the constants α_{3u} and $\alpha_{3\theta}$ the values of $\overline{u_2^2\theta}$ calculated from Eqs. (10) and (11) differ insignificantly. Therefore, it is difficult to give preference to the dynamic or the thermal scale on the basis of the three experiments analyzed.

The optimum values of the numerical coefficients in the models (6)-(11) for different types of flows are given in Table 2. It should be noted that the coefficients recommended in [4] for all the approximations having a dynamic scale prove to be smaller than those determined from the results of our experiments. Among the factors explaining this difference, we can point out the following, in particular. First, in [4] the optimum values of the coefficients were determined with allowance for different components of the tensor $\overline{u_iu_j\theta}$. In this case, as can be seen from the figures given in the paper, the optimum value of a numerical coefficient for approximating a concrete moment, $\overline{u_2^2\theta}$, for example, may differ considerably from the value recommended for all the components simultaneously. Second, the values of the coefficients can differ because of unequal errors in measuring the turbulence characteristics in these experiments. Thus, the error in measuring ε_u depends essentially on the finite resolving power of the filaments of the thermoanemometric probe and can result in almost twofold understatement of this quantity [7]. The fact that in [4], in contrast to the present paper, no correction of ε_u for the finite resolving power of the probe was made could lead to overstatement of the dynamic scale q^2/ε_u, and hence to a decrease in the coefficients in the approximations.

From the data presented in Table 2 it follows that the coefficients α_{3u} and $\alpha_{3\theta}$ depend on the type of flow to a considerably greater extent thant α_{2u} and $\alpha_{2\theta}$, although the model relations corresponding to them describe the experimental data more precisely. The components of the turbulent diffusion connected with the presence in the stream of the gradients of $\overline{u_iu_j}$ and $\overline{u_i\theta}$ and the average temperature gradient can be characterized, generally speaking, by different time scales. This fact is not taken into account at all in the approximations (10) and (11), which possibly leads to the dependence of the coefficients α_{3u} and $\alpha_{3\theta}$ on the type of flow.

For the practical application of the approximations under consideration, it must be verified whether their advantages are retained in the case when $\overline{u_2^2}$ is not taken from experiment

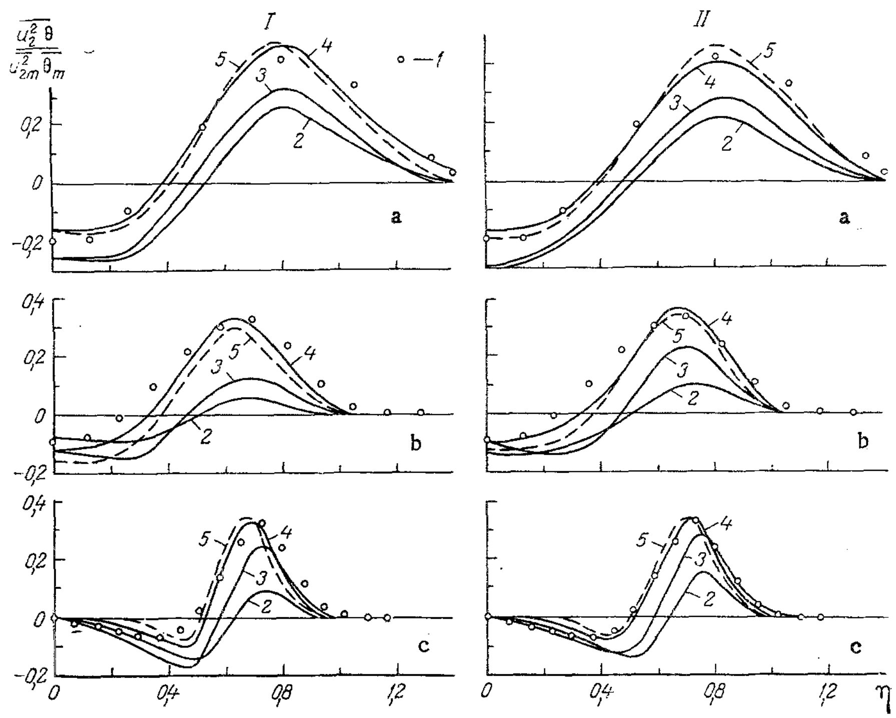

Fig. 2. Comparison of calculated and experimental values of $\overline{u_2^2\theta}$: a) axisymmetric wake ($J < 0$, $x_1/d = 50$); b) axisymmetric wake ($J = 0$, $x_1/d = 20$); c) plane wake ($x_1/d = 41$); 1) experiment; 2-5) calculation from approximations: I: 2) from Eq. (6); 3) (8); 4) (10); 5) (10) with $\overline{u_2^3}$ determined from (13); II: 2) from Eq. (7); 3) (9); 4) (11); 5) (11) with $\overline{u_2^3}$ determined from (13).

TABLE 2. Dependence of the Empirical Coefficients in the Models (6)-(11) on the Type of Flow

Flow	α_{1u}	α_{2u}	α_{3u}	α'_{3u}	$\alpha_{1\theta}$	$\alpha_{2\theta}$	$\alpha_{3\theta}$	$\alpha'_{3\theta}$
Axisymmetric wake ($J<0$)	0,10	0,10	0,064	0,066	0,29	0,30	0,18	0,20
Axisymmetric wake ($J=0$)	0,068	0,083	0,087	0,10	0,21	0,31	0,22	0,28
Plane wake ($J<0$)	0,077	0,11	0,046	0,042	0,24	0,27	0,11	0,11
Plane asymmetric heated jet ($J>0$) [4]	0,055	0,04	0,03	—	—	—	—	—

but calculated from some turbulence model. As was shown in [1], the best agreement with experimental data is provided by the model of Hanjalic and Launder:

$$\overline{u_iu_ju_k} = c\,\frac{q^2}{\varepsilon_u}\left(\overline{u_iu_e}\,\frac{\partial\overline{u_ju_k}}{\partial x_e} + \overline{u_ju_e}\,\frac{\partial\overline{u_iu_k}}{\partial x_e} + \overline{u_ku_e}\,\frac{\partial\overline{u_iu_j}}{\partial x_e}\right). \tag{12}$$

In the boundary-layer approximation we obtain the following model relation from (12):

$$\overline{u_2^3} = 3c\,\frac{q^2}{\varepsilon_u}\,\overline{u_2u_2}\,\frac{\partial\overline{u_2u_2}}{\partial x_2}. \tag{13}$$

The optimum value of the coefficient c, found from the condition of the smallest mean-square deviation of the calculated values of $\overline{u_2^3}$ from the results of the three experiments under consideration, is 0.08. The coefficient obtained differs little from that recommended in [1] ($c = 0.073$), which was determined on the basis of experimental investigations of velocity triple correlations in boundary and free flows. The use in Eqs. (10) and (11) of values of $\overline{u_2^3}$ found from Eq. (13) does not worsen the agreement between the calculated and experimental profiles of $\overline{u_2^2\theta}$ (see Fig. 2) and has slight infuence on the values of the empirical coefficients in Eqs. (10) and (11) (designated as α'_{3u} and $\alpha'_{3\theta}$ in Table 2).

Thus, the above analysis of different model relations showed that for good agreement between the calculated and experimental values of the moment $\overline{u_2^2\theta}$ the model must contain a term proportional to the average temperature gradient.

NOTATION

U_i, component of the average velocity in the direction x_i; u_i, component of the velocity pulsations; T, average temperature; θ, temperature pulsation; $\overline{u_i u_j}$, Reynolds stress; $\overline{u_i\theta}$, pulsation heat fluxes; $q^2 = \overline{u_i u_i}$, twice the kinetic energy of turbulence; $\varepsilon_u = \nu\overline{(\partial u_i/\partial x_k)^2}$, rate of dissipation of turbulent kinetic energy; $\varepsilon_\theta = \varkappa\overline{(\partial\theta/\partial x_k)^2}$, rate of "dissipation" of temperature pulsations; x_i, Cartesian coordinates (x_1 is longitudinal and x_2 is transverse); $\eta = x_2/\delta_\theta$, dimensionless coordinate; δ_θ, half-width of the wake; $\Delta T = T - T_\infty$, temperature defect; d, maximum transverse size of the body. Indices: 0, value of the characteristic at the wake axis; ∞, value in the undisturbed stream; m, maximum value; ( )', root-mean-square value; $\overline{(\ )}$, average value.

LITERATURE CITED

1. D. E. Cormack, L. G. Leal, and J. H. Seinfeld, Trans. ASME, J. Fluids Eng., 100, 47-54 (1978).
2. H. P. A. H. Irwin, J. Fluid Mech., 61, Part 1, 33-63 (1973).
3. E. P. Dyban, Processes of Turbulent Transfer in Reacting Systems [in Russian], Minsk (1985), pp. 55-64.
4. I. Dekeyser, J. Mech. Theor. Appl., 2, No. 6, 915-945 (1983).
5. Yu. M. Dmitrenko, V. L. Zhdanov, and N. N. Luchko, in: Heat and Mass Transfer: Results and Prospects [in Russian], Minsk (1985), pp. 60-62.
6. Yu. M. Dmitrenko, V. L. Zhdanov, N. N. Luchko, and P. Ya. Cherepanov, Inzh.-Fiz. Zh., 51, No. 6, 896-902 (1986).
7. Yu. M. Dmitrenko, V. L. Zhdanov, and B. A. Kolovandin, Inzh.-Fiz. Zh., 50, No. 1, 14-22 (1986).
8. E. Naudasher, J. Fluid Mech., 22, Part 4, 625-656 (1965).
9. P. Ya. Cherepanov, in: Processes of Turbulent Transfer [in Russian], Minsk (1985), pp. 56-74.
10. P. Ya. Cherepanov, in: Structure of Turbulent Flows [in Russian], Minsk (1982), pp. 132-143.
11. C. C. Alexopoulos and J. E. Keffer, Phys. Fluids, 14, No. 2, 216-224 (1971).
12. Yu. M. Dmitrenko, in: Structure of Turbulent Flows [in Russian], Minsk (1982), pp. 82-90.